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APPLICATION OF JUNGCK CONTRACTION IN FUZZY METRIC SPACE

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ABSTRACT

On the pattern of the most classical fixed point theorem. viz. the Banach contraction theorem, now the mathematicians are using Jungck contraction theorem for various applications. In this paper I proved some new theorems on fixed point which is further development of results of Tripathi and Mishra [8]

I. INTRODUCTION

The concept of fuzzy set is introduced by Zadeh [10] and then Kramosil and Michalek [7] defined fuzzy metric space by using of continuous t-norm. In 1999 Vasuki [9] proved some fixed point theorems in fuzzy metric space fpr R-weakly commuting mappings. Rhoades [5] gave an open problem " whether there exist a contractive definition which is strong to generate a fixed point but which does not force the map to be continuous at the fixed point". Balasubramaniam et al. [1] proved the open problem of Rhoades [5].

In 1986 Jungck [4] introduced the notion of compatible mapping and proved common fixed point for two mappings after this Singh and Jain [6] defined semi compatible mapping in fuzzy metric space and using this concept they proved common fixed point theorem for four self-mapping mapping on fuzzy metric space, further Tripathi et al. [8] improved the result of Singh and Jain

DEFINITION 1.1. [7] A binary operation $* : [0,1]\times[0,1]\to[0,1]$ is said to be continuous t–norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative.

(ii) $*$ is continuous.

(iii) $a * 1 = a$ for all $a\in[0,1]$.

$a*b \le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

DEFINITION 1.2. [7]. The three tuple $(X, M, *)$ is said to be fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2\times[0,\infty)$ satisfying the following conditions for all x, y, z in X and $s, t > 0$;

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(i) $M(x, y, 0) = 0,$

(ii) $M(x, y, t) = 1,$ if and only if $x = y,$

(iii) $M(x, y, t) = M(y, x, t),$

(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s),$

and (v) $M(x, y, \cdot) : [0, \infty) \to [0,1]$ is left continuous.

DEFINITION 1.3 [3]. Let (*X*, *M*, $*$) be a fuzzy metric space. A sequence $\{x_n\}$ in *X* is said to be convergent to a point x in X if

$$\lim_{n\to\infty} M(x_n, x, t) = 1$$

For all $t > 0.$

Further the sequence $\{x_n\}$ is said to be Cauchy sequence in X if

$$\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$$

for all $t > 0$ and $p > 0.$

The space (*X*, *M*, $*$) is said to be complete if every Cauchy sequence in it converges to a point of it.

In this paper, (*X*, *M*, $*$) is considered to be the fuzzy metric space with the condition,

$$\lim_{n\to\infty} M(x, y, t) = 1$$

for all x, y in $X.$ (1.1)

LEMMA 1.2 [2]. Let $\{y_n\}$ be a sequence in fuzzy metric space (*X*, *M*, $*$) with the condition (1.1). Suppose there exists a number $k \in (0,1)$ such that,

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t),$$

for all $t > 0.$

Then $\{y_n\}$ is a Cauchy sequence.

DEFINITION 1.4. [2] Let f and g be mappings from a fuzzy metric space (*X*, *M*, $*$) to itself. The mappings are said to be weak compatible if they commute at their coincidence points, that is,

fx = *gx* implies that *fgx* = *gfx.*

DEFINITION 1.5. [2] Let f and g be mappings from a fuzzy metric space (*X*, *M*, $*$) to itself. Then the mappings are said to be compatible if,

$$\lim_{n\to\infty} M(fgy_n, gfy_n, t) = 1, \quad \forall\ t > 0,$$

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whenever $\{y_n\}$ is a sequence in X such that, $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = x \in X$.

PROPOSITION 1.1 [4]. Self mappings f and g of a metric space $(X, M, *)$ are compatible, then they are weak compatible.

DEFINITION 1.6 [6]. Let f and g be mappings from a fuzzy metric space $(X, M, *)$ to itself. Then the mappings are said to be semi compatible if $\lim_{n\to\infty} M(fgy_n, gx, t) = 1, \ \forall\ t > 0,$ whenever $\{y_n\}$ is a sequence in X such that, $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = x \in X$.

Singh and Jain [6] proved that if the mappings f and g are semicompatible, then they are weak compatible without the converse being true.

II. MAIN RESULTS

THEOREM 2.1. Let $(X, M, *)$ be a fuzzy metric space and Y is an arbitrary set. Suppose $k \in (0,1)$ and $f, g, h, : Y \to X$ are mappings such that,

(i) $M(fx, gy, kt) \le \max\{M(hx, hy, t), M(fx, hx, t)\}$ $\forall$ $x, y \in Y$ and $t > 0$,

(ii) $f(Y) \cup g(Y) \subset h(Y)$,

and (iii) one of $f(Y)$, $g(Y), h(Y)$ is complete.

Then *f*, g and *h* have coincidence point.

PROOF. For $p_0 \in Y$ there exist $p_1, p_2 \in Y$ such that $fp_0 = hp_1, gp_1 = hp_2$ (because $f(Y) \cup g(Y) \subset h(Y)$). Inductively we can construct a sequence $\{p_n\}$ such that $fp_{2n} = hp_{2n+1},\ gp_{2n+1} = hp_{2n+2}$. Putting $x = p_{2n}$ and $y = p_{2n+1}$ in (i), we have

$$M(fp_{2n}, gp_{2n+1}, kt) \le \max\{M(hp_{2n}, hp_{2n+1}, t), M(fp_{2n}, hp_{2n}, t)\}$$

i.e. $M(hp_{2n+1}, gp_{2n+2}, kt) \le M(hp_{2n+1}, gp_{2n}, t)$.

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So by Lemma 1.1 $\{hp_n\}$ is a Cauchy sequence. Suppose $h(Y)$ is complete. Then $\{hp_n\} \to p \in h(Y).$ Also then there exists $u \in Y$ such that $hu = p.$ Putting $x = u, y = p_{2n+1}$ in (*i*), we have,

$$M(fu, gp_{2n+1}, kt) \le \max\{M(hu, hp_{2n+1}, t), M(fu, hu, t)\}.$$

Therefore, in limiting case as $n \to \infty,$

$$M(fu, p, kt) \le \max\{M(p, p, t), M(fu, hu, t)\}$$

i.e. $M(fu, p, kt) \le M(fu, p, t).$

Hence $f(u) = p = h(u).$ Lastly, putting $x = p_{2n+1},\ y = u$ in (*i*), we have

$$M(fp_{2n+1}, gu, kt) \le \max\{M(hp_{2n+1}, hu, t), M(fp_{2n+1}, hp_{2n+1}, t)\}$$

i.e. $M(p, gu, kt) \le M(p, p, t).$

Therefore $p = gu = f(u) = h(u).$ Hence *u* is coincidence point of *f*, g and *h*.

THEOREM 2.2. Let $(X, M, *)$ be a fuzzy metric space. Suppose $k \in (0,1)$ and $f, g, h,: X \to X$ are mappings such that,

(*i*) $M(fx, gy, kt) \le \max\{M(hx, hy, t), M(fx, hx, t)\ \ \forall\ x, y \in X$ and $t > 0,$

(*ii*) $f(X) \cup g(X) \subset h(X),$

(*iii*) one of $f(X)$, $g(X), h(X)$ is complete,

and (*iv*) f and h are coincidently commuting.

Then *f*, g and *h* have a unique common fixed point.

PROOF. In the theorem 2.1 if we take $Y = X$ then we get $p = gu = f(u) = h(u).$
Since *f* and *g* are coincidently commuting so,

$fhu = hfu$ and $fp = hp.$

Putting $x = fu,\ y = p_{2n+1}$ in (*i*), we have,

$$M(ffu, gp_{2n+1}, kt) \le \max\{M(hfu, hp_{2n+1}, t), M(ffu, hfu, t).$$

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Taking $n \to \infty$, we have, $M(ffu, p, kt) \le M(fp, p, t)$. Therefore $fp = hp = p$.

Again putting $x = p_{2n},\ y = p$ in (*i*) and taking limit as $n \to \infty$ we get, $gp = fp = hp = p$.

Hence *p* is a common fixed point of *f*, g and *h*.

For uniqueness suppose *p* and *q* are common fixed points of *f*, g and *h*.

Then by putting $x = p,\ y = q$ in (*i*) we get $p = q$. This proves the uniqueness

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